Olimpiada Naţionala de Matematică
Etapa locală, 11 februarie 2012
Clasa a IX-a

Barem de corectare clasa a IX-a

1. Ec. de forma $ax^2+bx+c=0, a=a_1$

A = {a_1, a_2 _ _ _ a_{51}}, $a_1 < a_2 <$ _ _ _ $< a_{51}$ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _1p

$\exists$ 50 numere diferite $a_2 - a_1$, $a_3 - a_1$ _ _ _ $a_{51} - a_1$ _ _ _ _ _ _ _ _ _ _ _ 1p

şi 51 numere diferite a_1, a_2 _ _ _ a_{51} _ 1p

Toate se afla in {1,2,3 _ _ _100}

=> $\exists$ k, l ai. $a_k - a_1 = a_l$ _ 1p

Luam b = a_k, c = a_l _1p

=> Δ = b^2 – 4ac = $(a+c)^2$ – 4ac = $(a-c)^2$ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _1p

=> $x_1, x_2 \in \mathbb{Q}$ _1p

2. Considera $a,b,c>0,\ x=\frac{a}{b},\quad y=\frac{b}{c},\quad z=\frac{c}{a}$ 2p

$$\frac{1+xy}{1+z}=\frac{1+\frac{a}{c}}{1+\frac{c}{a}}=\frac{a}{c}=\frac{a^2}{ac}\quad \text{si analoagele} \text{ ....}2p$$

$$\frac{a^2}{ac}+\frac{b^2}{ba}+\frac{c^2}{cb}\geq\frac{(a+b+c)^2}{ab+bc+ca} \text{ ....}2p$$

$$\frac{(a+b+c)^2}{ab+bc+ca}\geq 3 \text{ ....}1p$$

3 a) T. Bisectoarei: $\frac{A'B}{A'C}=\frac{c}{b} \Rightarrow A'B=\frac{ac}{b+c}$

I. $A'B<\frac{a}{2} \Leftrightarrow b>c$ _1 p

T. Menelaus in Δ ABA’ transversala E – I – D

$\frac{AE}{EB}\cdot\frac{BD}{DA'}\cdot\frac{A'I}{IA}=1$ _ 1p

T. bisectoarei in Δ ABA’: $\frac{A'I}{IA}=\frac{BA'}{BA}=\frac{a}{b+c}$

$DA'=\frac{a(b-c)}{2(b+c)}$ _ 1 p

$\frac{AE}{EB}=\frac{b-c}{a} => \frac{AE}{EB}=\frac{b-c}{a+b-c}$ _1p

II. $A'B>\frac{a}{2} <=> b>c$ _ 1p

Analog caz $I => \frac{AE}{EB}=\frac{c-b}{a+b-c}$ _1p

b) Conditia: b > c (cazul I) _1p

4. Fie M $\in$ (ABC) a.i. $\exists$ α $\in$ R cu $\overrightarrow{MA}+\overrightarrow{MB}+2\overrightarrow{MC}=\alpha\overrightarrow{AB}$ _ _ _ _ _ _ _ _ _ _ _1p

{D} = mijlocul segmentului [AC]
{E} = mijlocul segmentului [BC]

$\overrightarrow{MA}+\overrightarrow{MC}=2\overrightarrow{MD}$ si $\overrightarrow{MB}+\overrightarrow{MC}=2\overrightarrow{ME}$

=> $2\overrightarrow{MD}+2\overrightarrow{ME}=\alpha\overrightarrow{AB}=2\alpha\overrightarrow{DE}$ _ 1p

$\overrightarrow{MD}+\overrightarrow{ME}=\alpha(\overrightarrow{ME}-\overrightarrow{MD})$

$(1+\alpha)\overrightarrow{MD}=(1-\alpha)\overrightarrow{ME}$ _ 1p

Daca α = -1 => M = E

Daca α = 1 => M = D...1P

Daca α $\in$ R\{-1,1} => $\overrightarrow{MD}$ si $\overrightarrow{ME}$ coliniari => M $\in$ DE_ _ _ _ _ _ _ _ _ _ _ 1P

Reciproc, dc. M $\in$ DE => $\overrightarrow{MA}+\overrightarrow{MC}+\overrightarrow{MB}+\overrightarrow{MC}=2(\overrightarrow{MD}+\overrightarrow{ME})=\beta\overrightarrow{DE}$, $\beta\in R^*$ _ 1p

$\overrightarrow{DE}=\frac{1}{2}\overrightarrow{AB} => \overrightarrow{MA}+\overrightarrow{MB}+2\overrightarrow{MC}$ coliniar cu AB_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _1p