

Olimpiada Națională de Matematică  
Etapa locală, 11 februarie 2012  
Clasa a IX-a

Barem de corectare clasa a IX-a

1. Ec. de forma  $ax^2 + bx + c = 0, a = a_1$   
 $A = \{a_1, a_2, \dots, a_{51}\}, a_1 < a_2 < \dots < a_{51}$  .....1p  
 $\exists$  50 numere diferite  $a_2 - a_1, a_3 - a_1, \dots, a_{51} - a_1$  .....1p  
și 51 numere diferite  $a_1, a_2, \dots, a_{51}$  .....1p  
Toate se afla in  $\{1, 2, 3, \dots, 100\}$   
 $\Rightarrow \exists k, l$  ai.  $a_k - a_1 = a_l$  .....1p  
Luam  $b = a_k, c = a_l$  .....1p  
 $\Rightarrow \Delta = b^2 - 4ac = (a + c)^2 - 4ac = (a - c)^2$  .....1p  
 $\Rightarrow x_1, x_2 \in \mathbb{Q}$  .....1p

2. Considera  $a, b, c > 0, x = \frac{a}{b}, y = \frac{b}{c}, z = \frac{c}{a}$  .....2p

$$\frac{1+xy}{1+z} = \frac{1+\frac{a}{c}}{1+\frac{c}{a}} = \frac{a}{c} = \frac{a^2}{ac}$$

si analoagele .....2p

$$\frac{a^2}{ac} + \frac{b^2}{ba} + \frac{c^2}{cb} \geq \frac{(a+b+c)^2}{ab+bc+ca}$$
 .....2p

$$\frac{(a+b+c)^2}{ab+bc+ca} \geq 3$$
 .....1p

- 3 a) T. Bisectoarei:  $\frac{A'B}{A'C} = \frac{c}{b} \Rightarrow A'B = \frac{ac}{b+c}$

I.  $A'B < \frac{a}{2} \Leftrightarrow b > c$  .....1p

T. Menelaus in  $\Delta ABA'$  transversala E – I – D

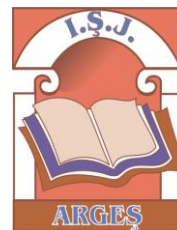
$$\frac{AE}{EB} \cdot \frac{BD}{DA'} \cdot \frac{A'I}{IA} = 1$$
 .....1p



# ROMÂNIA

MINISTERUL EDUCAȚIEI, CERCETĂRII,  
TINERETULUI ȘI SPORTULUI

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T. bisectoarei in  $\Delta ABA'$ :  $\frac{A'I}{IA} = \frac{BA'}{BA} = \frac{a}{b+c}$

$$DA' = \frac{a(b-c)}{2(b+c)} \text{-----} 1 \text{ p}$$

$$\frac{AE}{EB} = \frac{b-c}{a} \Rightarrow \frac{AE}{EB} = \frac{b-c}{a+b-c} \text{-----} 1 \text{ p}$$

II.  $A'B > \frac{a}{2} \Leftrightarrow b > c$  ----- 1 p

Analog caz  $I \Rightarrow \frac{AE}{EB} = \frac{c-b}{a+b-c}$  ----- 1 p

b) Condiția:  $b > c$  (cazul I) ----- 1 p

4. Fie  $M \in (ABC)$  a.i.  $\exists \alpha \in \mathbb{R}$  cu  $\vec{MA} + \vec{MB} + 2\vec{MC} = \alpha\vec{AB}$  ----- 1 p

{D} = mijlocul segmentului [AC]

{E} = mijlocul segmentului [BC]

$$\vec{MA} + \vec{MC} = 2\vec{MD} \text{ si } \vec{MB} + \vec{MC} = 2\vec{ME}$$

$$\Rightarrow 2\vec{MD} + 2\vec{ME} = \alpha\vec{AB} = 2\alpha\vec{DE} \text{-----} 1 \text{ p}$$

$$\vec{MD} + \vec{ME} = \alpha(\vec{ME} - \vec{MD})$$

$$(1 + \alpha)\vec{MD} = (1 - \alpha)\vec{ME} \text{-----} 1 \text{ p}$$

Daca  $\alpha = -1 \Rightarrow M = E$

Daca  $\alpha = 1 \Rightarrow M = D$ ..... 1 P

Daca  $\alpha \in \mathbb{R} \setminus \{-1, 1\} \Rightarrow \vec{MD}$  si  $\vec{ME}$  coliniari  $\Rightarrow M \in DE$  ----- 1 P

Reciproc, dc.  $M \in DE \Rightarrow \vec{MA} + \vec{MC} + \vec{MB} + \vec{MC} = 2(\vec{MD} + \vec{ME}) = \beta\vec{DE}$ ,  $\beta \in \mathbb{R}^*$  ----- 1 p

$$\vec{DE} = \frac{1}{2}\vec{AB} \Rightarrow \vec{MA} + \vec{MB} + 2\vec{MC} \text{ coliniar cu } AB \text{-----} 1 \text{ p}$$